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Non-classicality of photon added coherent and thermal radiations

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Abstract. Production and analysis of non-Gaussian radiation fields has evinced a lot of attention recently. Simplest way of generating such non-Gaussians is through adding (subtracting) photons to Gaussian fields. Interestingly, when photons are added to classical Gaussian fields, the resulting states exhibit *non-classicality*. Two important classical Gaussian radiation fields are coherent and thermal states. Here, we study the non-classical features of such states when photons are added to them. Non-classicality of these states shows up in the negativity of the Wigner function. We also work out the *entanglement potential*, a recently proposed measure of non-classicality for these states. Our analysis reveals that photon added coherent states are non-classical for all seed beam intensities; their non-classicality increases with the addition of more number of photons. Thermal state exhibits non-classicality at all temperatures, when a photon is added; lower the temperature, higher is their non-classicality.

PACS. 42.50.Dv Nonclassical states of the electromagnetic field, including entangled photon states; quantum state engineering and measurements – 03.65.Wj State reconstruction, quantum tomography – 03.67.Mn Entanglement production, characterization, and manipulation

QICS. 02.10.+t Quantum-Classical Transition – 01.30.+r Quantum states and dynamics as a resource for information processing

1 Introduction

Several branches of quantum optics from non-linear optics to laser-physics and cavity QED are very actively engaged in a variety of processes producing non-classical light. Such radiation fields attract attention, not only because they provide a platform for testing fundamental concepts of quantum theory, but also for applications of importance like precision measurements in interferometry [1]. Moreover, rapidly developing area of quantum computation and information theory has kindled further interest in generating and manipulating non-classical radiation fields — called quantum continuous variable states. These states are promising candidates for many applications of quantum information technology [2,3]. In such a context, Gaussian light fields gain prominence, both in view of their conceptual and experimental importance. However, a need to leap beyond Gaussian domain has been emphasized [4] and the degaussification process has been catching a lot of interest. Degaussification can be realized in a simple manner by adding (subtracting) photons to (from) a Gaussian field and the resulting states are known to exhibit non-classical properties such as negativity of the Wigner function [5], antibunching [6], sub-Poissonian photon statistics [7] or squeezing in one of the quadratures of the field [8], etc.

Almost a decade ago Agarwal and Tara [9] introduced, theoretically, a new class of non-Gaussian states, which is obtained by repeated application of the photon creation operator on the coherent state. The resulting class of states were identified to lie between the Fock state and the coherent state and were indeed non-classical. Recently, single photon excitation of a classical coherent field has been generated experimentally [10] and ultrafast, time-domain, quantum homodyne tomography technique has explicitly demonstrated a quantum to classical transition. In another development, a traveling non-Gaussian field was experimentally produced by subtracting a photon from a squeezed vacuum [4]. While the pulsed homodyne detection scheme confirmed non-Gaussian statistics for the photon subtracted squeezed vacuum, the Wigner function reconstructed from the experimental data failed to exhibit negativity. Kim et al. [11] analyzed the non-classicality of photon subtracted Gaussian fields and identified that the photo detection efficiency as well as the modal purity parameter were not high enough to record a negative Wigner function in the experiment [4] of Wenger et al. Moreover, Kim et al. show that unless the input

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Gaussian radiation is non-classical, one cannot generate a non-classical field through photon subtraction [11]. They also point out the contrasting situation of photon addition to Gaussian fields, where even a highly classical state like thermal state turns out to be non-classical [12,13]. It may be noted that photon addition to a thermal state results in the removal of the vacuum part [13] and hence leads to a *truncated thermal state*. All such *truncated states*, with their vacuum contribution removed, are shown to be non-classical by Lee [12].

In view of the current experimental progress [4,10] in the production of non-Gaussian radiation fields, and also in the tomographic reconstruction [4,10,14] of Wigner functions of quantum states, it is timely to analyze the non-classicality of photon added classical radiation fields, through negativity of their Wigner functions. In this paper, we investigate photon added coherent and thermal states.

Entanglement, another striking quantum feature, has occupied a central position in the development of quantum information processing [2,3]. There has been a considerable progress in understanding the connection between non-classicality and entanglement. It has been identified [15,16] that non-classicality is an unmistakable source of entanglement. A beam splitter is capable of converting non-classicality of a single mode radiation into bipartite entanglement. This property viz., *non-classicality as an entanglement resource*, has been employed recently [15], to identify *Entanglement potential* (EP) — a computable measure of non-classicality — of single mode radiation fields. EP allows us to analyze the degree of non-classicality of a given single mode radiation field. We compute EP of photon added coherent states (PACS) and show that the EP reduces with the increase of seed beam intensity, which is in confirmation with the analysis of the Wigner function of the state. A comparison of EP's of single, two and three photon added coherent states reveals that non-classicality of PACS increases with the addition of more number of photons. We verify that the Wigner function of a photon added thermal state is negative at the phase space origin for all temperatures, while the EP of the state, evaluated in the low temperature limit, reveals that non-classicality of the state reduces with increasing temperature.

2 Measures of non-classicality

Generally a non-classical state is recognized as one, which cannot be written as a statistical mixture of coherent states. It has been well accepted that the non-existence of a well defined Glauber-Sudarshan P-function [17] implies *non-classicality* of a given state. However this identification poses operational difficulties, as it requires complete information of the state to be examined, so that it's P-function can be reconstructed. Several operational criteria, which are equivalent to the one based on the P-function and which can be used to distinguish between classical and non-classical states in experimental measurements have been proposed from the early days of quantum optics. Such signatures of non-classicality, verifiable in a simple experiment are, antibunching [6] and sub-Poissonian photon statistics [7], squeezing [8], photon number oscillations [18], negative value of Wigner function [5], etc.

Here, we focus our attention on the Wigner function of a given quantum state. The non-classicality character of a state is strongly registered by negativity of the Wigner function. Especially, Fock states show a negative dip around the phase space origin, as has been clearly reflected in the experimentally reconstructed Wigner function [14]. Moreover, there has been an ongoing effort towards more efficient quantum homodyne tomographic techniques [4,10,14], and in such a context, analysis of Wigner function proves to be useful.

The Wigner function of a system, characterized by the density operator ρ is defined [5] through

$$W(q,p) = \frac{1}{\pi}\int \langle q+y|\rho|q-y\rangle e^{-2ipy}dy. \qquad (1)$$

Basically Wigner function is a quasi-probability distribution representing quantum states in phase space. It is not a true probability distribution as it can take negative values also. If, for a state, Wigner function takes negative value, the quantum state has no classical analog. However, the converse does not hold good: when the Wigner function is positive everywhere, one can not conclude that the state is classical. For example, for a squeezed state, Wigner function is a Gaussian and is positive throughout. But, squeezed radiation [8] is one of the most important non-classical field. Thus, one has to resort to other measures of non-classicality.

There have been several approaches to quantify non-classicality of a state through universal measures like, Hillery's non-classical distance [19] and Lee's non-classical depth [20]. Distance between a given non-classical state and the set of all classical states is non-zero and hence, serves as a measure of non-classicality, called *non-classical distance* [19]. However, identifying an optimal reference classical state is one of the main problems associated with these distance based measures [21] of non-classicality.

Lee's [20] non-classical depth, $0 \leq \tau_m \leq 1$, is essentially the quantity of smoothing required to transform a non-positive P-function into a well behaved positive distribution. (Non-classical depth τ_m is also interpreted as the minimum average number of thermal photons that are necessary to destroy the non-classical effects of a given state). A classical state has $\tau_m = 0$. For a pure Gaussian state (Squeezed vacuum), the non-classical depth varies between 0 and 1/2 [20]; for all non-Gaussian pure states $\tau_m = 1$ [22], thus placing all such states to be identical, as far as their non-classicality is concerned. Therefore, this criterion forces one to conclude that the non-classicality of PACS is independent of the seed beam intensity. Moreover, non-classical depth of photon added thermal states (for that matter, all *truncated states*) is shown to be a maximum i.e., $\tau_m = 1$ [13]. According to this measure, photon added thermal states are equally (maximally) non-classical at all temperatures.

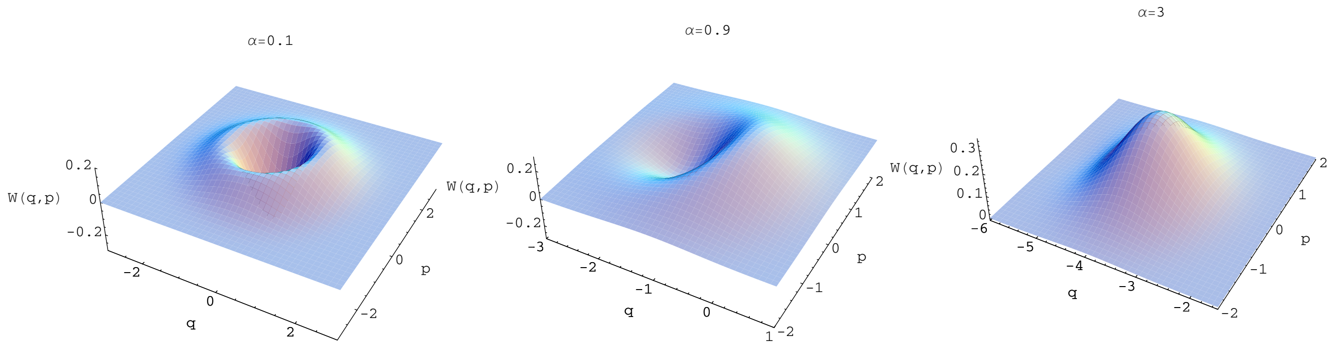

Fig. 1. (Color online) Wigner function of SPACS for different beam intensities $|\alpha|^2$. Here, (a) $\alpha = 0.1$ (b) for $\alpha = 0.9$ and (c) for $\alpha = 3$.

In this paper, we consider Entanglement potential (EP) [15], as a universal measure of non-classicality for our discussion of PACS and photon added thermal states. EP gives the amount of two-mode entanglement that can be generated from a non-classical input state, in a linear optics set up. It is important to note here that a classical single mode radiation *does not* get entangled in such an arrangement [16]. EP is nothing but the logarithmic negativity [23] of a bipartite quantum state ϱ_σ, which results from mixing a given single mode state σ, with vacuum state, in a 50:50 beam splitter. More specifically, EP is defined as

$$\mathrm{EP} = \log_2 \|\varrho_\sigma^{PT}\|_1, \tag{2}$$

where ϱ_σ^{PT} denotes the partial transpose of a two-mode density operator $\rho_\sigma = U_{BS}(\sigma \otimes |0\rangle\langle 0|)U_{BS}^\dagger$. In equation (2), the symbol $\|\cdot\|_1$ denotes the trace norm[1] and U_{BS} corresponds to a 50:50 beam splitter i.e., $U_{BS} = \exp(\frac{\pi}{2}(a^\dagger b - ab^\dagger))$, whose action (Heisenberg view point) on the creation operators $a^\dagger, b^\dagger$ of the two input ports is explicitly given by

$$\begin{aligned} U_{BS} a^\dagger U_{BS} &= (a^\dagger + b^\dagger)/\sqrt{2} \\ U_{BS} b^\dagger U_{BS} &= (b^\dagger - a^\dagger)/\sqrt{2}. \end{aligned} \tag{3}$$

The state σ is said to be non-classical, iff its entanglement potential is nonzero. EP has been evaluated [15] for a variety of non-classical states like squeezed states, even and odd coherent states, Fock states, etc.

The remainder of this paper is devoted to a study on photon addition to (i) coherent state, an example of a pure Gaussian state and (ii) thermal state, a mixed Gaussian state, both of which are well-known classical states. Action of photon creation operator on these states results in non-classical, non-Gaussian states. We study the non-classicality of these states through negativity of the Wigner function and the entanglement potential.

[1] Trace norm of a partially transposed density operator ϱ^{PT} is given by $\|\varrho^{PT}\|_1 = 1 + 2N(\rho)$, where the negativity $N(\rho)$ is the sum $|\sum_i \lambda_i|$ of all the negative eigenvalues of ϱ^{PT}.

3 Photon added coherent state

Coherent states are the analogs of classical radiation fields. These states are described by a Poissonian photon number distribution and have a well defined amplitude and phase. It is interesting to see how these states turn non-classical, when a single quantum of radiation excites them.

Recently [10], single photon added coherent states [SPACS] has been generated experimentally and tomographically reconstructed Wigner function for such states has been analyzed. SPACS are obtained by application of creation operator $a^\dagger$ on a coherent state $|\alpha\rangle$. Normalized SPACS is given by

$$\begin{aligned} |\mathrm{SPACS}\rangle = \frac{a^\dagger|\alpha\rangle}{(1+|\alpha|^2)} &= \frac{a^\dagger D_a(\alpha)|0_a\rangle}{\sqrt{(1+|\alpha|^2)}} \\ &= \frac{D_a(\alpha)[|1_a\rangle + \alpha^*|0_a\rangle]}{\sqrt{(1+|\alpha|^2)}}, \end{aligned} \tag{4}$$

where $D_a(\alpha) = \exp(a^\dagger\alpha - a\alpha^*)$ is the displacement operator and the coherent state $|\alpha\rangle = D_a(\alpha)|0_a\rangle$; $|0_a\rangle$ denotes the vacuum state. Here we have used the property, $D_a^\dagger(\alpha)a^\dagger D_a(\alpha) = a^\dagger + \alpha^*$. Wigner function of a SPACS is given by [9]

$$W(q,p) = \frac{-L_1(|2c-\alpha|^2)}{\pi L_1(-|\alpha|^2)}\exp(-2|c-\alpha|^2), \tag{5}$$

where $c = (q+ip)/\sqrt{2}$, and $L_1(z) = 1 - z$ is Laguerre polynomial of first order. Note that $W(q,p)$ of the SPACS is negative when $|2c-\alpha|^2 < 1$.

In Figure 1 we have plotted the Wigner function of SPACS, for various beam intensities $|\alpha|^2$. It is clear from these plots that negativity of Wigner function reduces with the increase of the intensity $|\alpha^2|$.

We may verify whether addition of more number of photons leads to higher non-classicality of the beam. To see this we consider two photon added coherent states. Wigner function of such states is explicitly given by,

$$W(q,p) = \frac{L_2(|2c-\alpha|^2)}{\pi L_2(-|\alpha|^2)}\exp(-2|c-\alpha|^2), \tag{6}$$

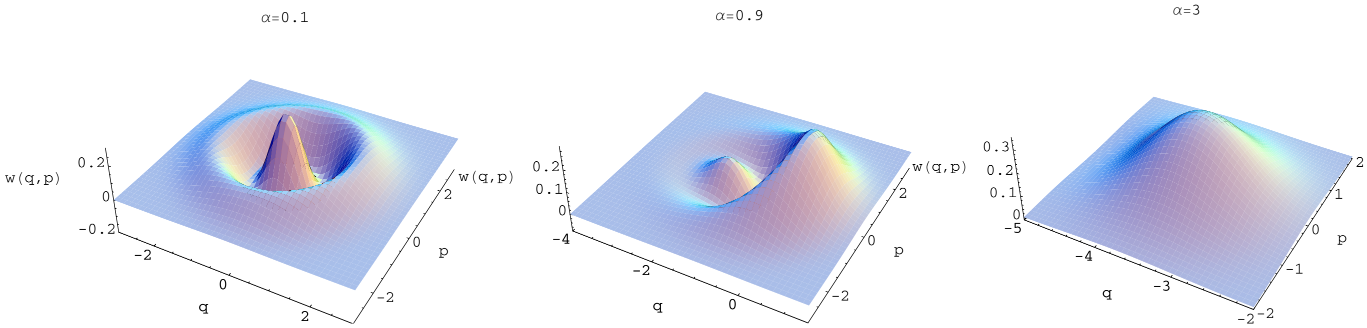

Fig. 2. (Color online) Wigner function of two PACS for different beam intensities $|\alpha|^2$. Here, (a) $\alpha = 0.1$ (b) for $\alpha = 0.9$ and (c) for $\alpha = 3$.

where $L_2(z) = 1 - 2z + z^2/2$ is Laguerre polynomial of second order.

Figure 2 gives the plots of the Wigner functions of two photon added coherent state for varying seed beam intensities. We note the same behavior here too, viz., the non-classicalities — depicted through the Wigner functions — decrease with increasing intensity $|\alpha|^2$. But through these plots we can not conclude if addition of more number of photons leads to higher non-classicality or not.

To investigate this, we now evaluate the EP of these photon added states. Let us first consider a SPACS. When SPACS is mixed with vacuum state $|0\rangle$ and is sent through a 50:50 beam splitter, the resulting two mode state is given by

$$|\psi\rangle = U_{BS}(|\text{SPACS}\rangle \otimes |0\rangle) = D_a\left(\frac{\alpha}{\sqrt{2}}\right) D_b\left(\frac{\alpha}{\sqrt{2}}\right) \times \left[\frac{|1_a 0_b\rangle + |0_a 1_b\rangle + \sqrt{2}\alpha^* |0_a 0_b\rangle}{\sqrt{2(1+|\alpha|^2)}}\right], \quad (7)$$

since a 50:50 beam splitter U_{BS} acts on $D_a(\alpha)$ as (see Eq. (3))

$$U_{BS} D_a(\alpha) U_{BS}^\dagger = D_a\left(\frac{\alpha}{\sqrt{2}}\right) D_b\left(\frac{\alpha}{\sqrt{2}}\right), \quad (8)$$

with $D_a(\alpha/\sqrt{2}) = \exp((a^\dagger\alpha - a\alpha^*)/\sqrt{2})$ and $D_b(\alpha/\sqrt{2}) = \exp((b^\dagger\alpha - b\alpha^*)/\sqrt{2})$. The corresponding two mode density operator is given by

$$\begin{aligned}\varrho_0' &= |\psi\rangle\langle\psi| \\ &= D_a\left(\frac{\alpha}{\sqrt{2}}\right) D_b\left(\frac{\alpha}{\sqrt{2}}\right) \varrho_0 D_a^\dagger\left(\frac{\alpha}{\sqrt{2}}\right) D_b^\dagger\left(\frac{\alpha}{\sqrt{2}}\right)\end{aligned} \quad (9)$$

with

$$\varrho_0 = \frac{1}{2(1+|\alpha|^2)} \begin{pmatrix} 2|\alpha|^2 & \sqrt{2}\alpha^* & \sqrt{2}\alpha^* & 0 & \cdot & \cdot \\ \sqrt{2}\alpha & 1 & 1 & 0 & \cdot & \cdot \\ \sqrt{2}\alpha & 1 & 1 & 0 & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} \quad (10)$$

in the Fock state basis $|n_a m_b\rangle$; n_a, $m_b = 0, 1, 2, ...$ Since ϱ_0' is locally equivalent to ϱ_0(as$D_a\left(\alpha/\sqrt{2}\right) D_b\left(\alpha/\sqrt{2}\right)$ corresponds to a local displacements on the two mode states), EP of ϱ_0' and that of ϱ_0 are same. So we proceed with the evaluation of EP of the state ϱ_0 itself.

The partial transpose of ϱ_0 is given by

$$\varrho_0^{\text{PT}} = \frac{1}{2(1+|\alpha|^2)} \begin{pmatrix} 2|\alpha|^2 & \sqrt{2}\alpha & \sqrt{2}\alpha^* & 1 & 0 & \cdot & \cdot \\ \sqrt{2}\alpha^* & 1 & 0 & 0 & 0 & \cdot & \cdot \\ \sqrt{2}\alpha & 0 & 1 & 0 & 0 & \cdot & \cdot \\ 1 & 0 & 0 & 0 & 0 & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}, \quad (11)$$

where, except for the first 4×4 diagonal block all the other elements are zero. The non-zero eigenvalues of ϱ_0^{PT} are easily identified to be,

$$\begin{aligned}\lambda_1 &= \frac{1}{2(1+|\alpha|^2)}, \qquad \lambda_2 = -\frac{1}{2(1+|\alpha|^2)}, \\ \lambda_{3,4} &= \frac{(1+|\alpha|^2) \pm \sqrt{(1+|\alpha|^2)^2 - 1)}}{2(1+|\alpha|^2)}.\end{aligned} \quad (12)$$

Note that λ_2 is the only negative eigenvalue of ϱ^{PT}, and hence, the EP of a SPACS is given by

$$\begin{aligned}\text{EP} &= \log_2 \|\varrho_0^{\text{PT}}\|_1 = \log_2(1 + 2|\lambda_2|) \\ &= \log_2\left(\frac{2+|\alpha|^2}{1+|\alpha|^2}\right).\end{aligned} \quad (13)$$

Following similar lines, we can evaluate entanglement potentials of two and three photon added coherent states also. But the expressions are lengthy and do not exhibit a simple structure. We have computed them numerically and plots of entanglement potential for single, two and three photon added coherent states are given in Figure 3. From the figure it is evident that EP is non zero for low intensity of the seed beam and it reduces gradually with the increase of intensity showing that the state is non-classical for all seed beam intensities. This observation is in confirmation with the conclusions reached through Wigner

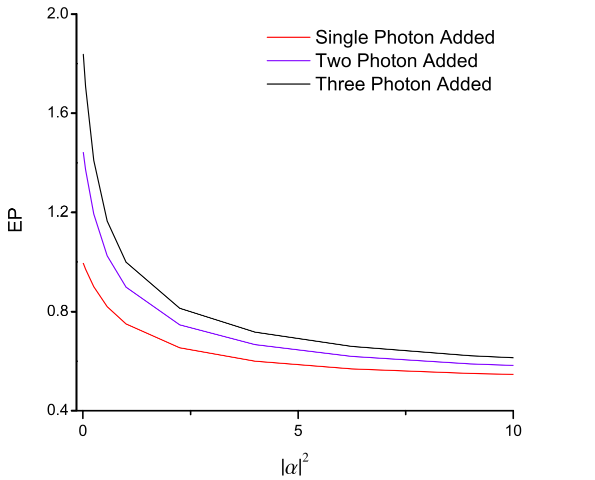

Fig. 3. (Color online) Entanglement potential for single, two and three photon added coherent states.

function analysis. While the EPs of single, two and three photon added coherent states converge for higher beam intensities, they are all different for low beam intensity, with larger value for higher photon added states. This observation reveals that non-classicality of photon added coherent state increases with the addition of larger number of photons.

4 Photon added thermal state

Density matrix of single mode thermal state of system in thermal equilibrium, characterized by the Hamiltonian $\hat{H} = a^\dagger a\hbar\omega$ is given by

$$\rho_{\rm th} = \frac{\exp\left(-\frac{\hat{H}}{kT}\right)}{{\rm Tr}\left[\exp\left(-\frac{\hat{H}}{kT}\right)\right]}. \tag{14}$$

In the Fock state basis, $\rho_{\rm th}$ can be expressed in the form

$$\rho_{\rm th} = A\sum_{n=0}^{\infty} x^n|n\rangle\langle n|, \tag{15}$$

where $A = 1-x$, $x = e^{-\frac{\hbar\omega}{kT}}$; $0 \leq x \leq 1$. Note that $x \to 0$ limit corresponds to $T \to 0$ and $x \to 1$ implies $T \to \infty$. Photon added thermal state is obtained through the application of creation operator on the thermal state i.e.,

$$\begin{aligned} a^\dagger\rho_{\rm th}a &= (1-x)\sum_{n=0}^{\infty} x^n a^\dagger|n\rangle\langle n|a \\ &= (1-x)\sum_{n=0}^{\infty} x^n(n+1)|n+1\rangle\langle n+1|. \end{aligned} \tag{16}$$

Simplifying equation (16) we get,

$$\begin{aligned} a^\dagger\rho_{\rm th}a &= (1-x)\sum_{m=0}^{\infty} m x^{m-1}|m\rangle\langle m| \\ &= (1-x)\frac{\partial}{\partial x}\left(\frac{1}{1-x}\rho_{\rm th}\right). \end{aligned} \tag{17}$$

Normalized photon added thermal state is given by

$$\rho^{pa}_{\rm th} = (1-x)^2\frac{\partial}{\partial x}\left(\frac{1}{1-x}\rho_{\rm th}\right). \tag{18}$$

Wigner function of a thermal state has been identified [5] to be

$$W_{\rm th}(q,p) = \frac{1}{\pi}B\exp[-B(q^2+p^2)];\ B = \frac{1-x}{1+x}. \tag{19}$$

Making use of the above equation, it is easy to evaluate the Wigner function of a photon added thermal state:

$$W^{pa}_{\rm th}(q,p) = (1-x)^2\frac{\partial}{\partial x}\left(\frac{1}{1-x}W_{\rm th}(q,p)\right). \tag{20}$$

After simplification, we get the Wigner function of the photon added thermal state as

$$W^{pa}_{\rm th}(q,p) = \frac{1}{\pi}B^2\left[\frac{2(q^2+p^2)}{(1+x)} - 1\right]\exp[-B(q^2+p^2)]. \tag{21}$$

It is clear that the Wigner function W^{pa}_{th} is negative at the origin of phase space, at all temperatures. We have plotted the Wigner function for different values of the parameter x — which in turn corresponds to various temperatures — in Figure 4. It is clear from the plots that the photon added thermal states at various temperature are all non-classical.

Entanglement potential of photon added thermal states can be evaluated in the low temperature limit, since the higher Fock states have lesser occupancy in this limit leading to the truncation of the Hilbert space. Retaining terms up to first and second order in the parameter x, the density matrices of photon added thermal state are given below:

$$\begin{aligned} &\text{I order in } x\text{: } \rho^{pa}_{th} = (1-2x)|1\rangle\langle 1| + 2x|2\rangle\langle 2| \\ &\text{II order in } x\text{: } \rho^{pa}_{th} = \frac{1}{(1-x^2)}[(1-2x)|1\rangle\langle 1| \\ &\qquad +2x(1-2x)|2\rangle\langle 2| + 3x^2|3\rangle\langle 3|]. \end{aligned} \tag{22}$$

Following similar steps to evaluate EP of SPACS, we can numerically compute the entanglement potentials of photon added thermal states too, in the low temperature limit. With the help of this analysis we realize that EP reduces with increasing temperature. We therefore conclude that non-classicality of photon added thermal states reduces gradually with the increase of temperature.

In conclusion, we have analyzed the non-classicality of photon added coherent states and thermal states, using

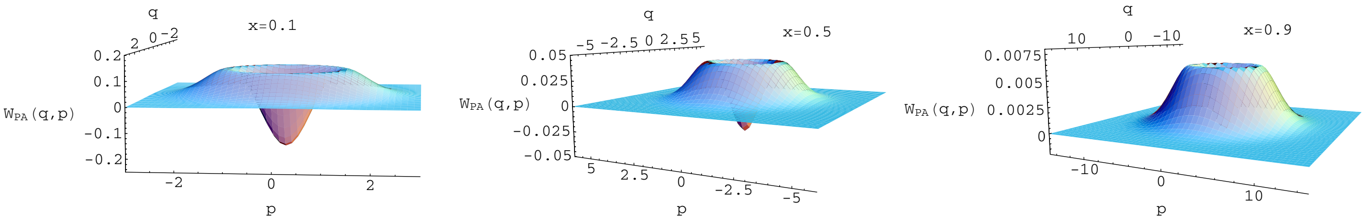

Fig. 4. (Color online) Wigner functions of single photon added thermal state for various temperatures: (a) $x = 0.1$ (b) $x = 0.5$ (c) $x = 0.9$.

(i) negativity of the Wigner function and (ii) entanglement potential. We have shown that photon added coherent states are non-classical for all seed beam intensities; the degree of non-classicality reduces with the increase of intensity. Photon added thermal states are shown to be non-classical at all temperatures and their non-classicality reduces with the increase of temperature.

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